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Momentum distributions for the quantum δ-kicked rotor with decoherence

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We report on the momentum distribution line shapes for the quantum δ-kicked rotor in the presence of environment induced decoherence. Experimental and numerical results are presented. In the experiment ultracold cesium atoms are subjected to a pulsed standing wave of near resonant light. Spontaneous scattering of photons destroys dynamical localization. For the scattering rates used in our experiment the momentum distribution shapes remain essentially exponential.

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Environment induced decoherence plays a significant role in contemporary quantum physics through its potential for explaining long-standing problems in measurement theory. Any real, open quantum system leaks coherence to its surroundings via extraneous degrees of freedom, which are coupled to the environment [1,2]. Dynamical localization in the quantum δ-kicked rotor (Q-DKR) has been examined theoretically [3], and experimentally through its atomic optics manifestation [4]. Environment induced decoherence in the Q-DKR has been addressed in various theoretical studies [5–7], and through recent experiments utilizing ultracold cesium atoms [8–11].

The purpose of this Brief Report is to provide details on the measured and calculated momentum distributions for the Q-DKR in the presence of decoherence that were not presented in our recent publication [8]. It is also important to make comparisons with the results of the Austin group [10,11]. In our previous publication [8], we noted that for the spontaneous emission rates used in our experiment, the momentum distribution shapes remained exponential. The Austin group reports line shapes that diverge from exponential localization. It is important to note, however, that the two results are not necessarily inconsistent. The Austin group's spontaneous emission rate per kick (13%) was greatly in excess of our highest rate of 4.6%. At the relatively low spontaneous emission rates used in our experiment the momentum distribution remains essentially exponential, and delocalization reveals itself via energy diffusion rather than a transition from exponential to Gaussian (or some other) line shapes. A similar behavior has been found in the theoretical investigation of a phase modulated potential [12].

The details of our experimental system were described in our previous publications [8,9], but we will briefly review the key details. We consider an atom (transition frequency ω_0) suspended in a standing wave of near resonant light (frequency ω_l or wavenumber k_l). Since the degradation of dynamical localization does not require a large amount of spontaneous emission, the atomic dynamics will be predominantly coherent. We therefore neglect spontaneous emission in the initial formulation. Under the assumption of large detuning compared to the Rabi frequency, the resulting Hamiltonian governing the coherent time evolution is

$$H=\frac{p^2}{2m}-\frac{\hbar\Omega_{eff}}{8}\cos(2k_lx)\sum_{q=1}^{N}f(t-gT), \tag{1}$$

where $\Omega_{eff}=\Omega^2(s_{45}/\delta_{45}+s_{44}/\delta_{44}+s_{43}/\delta_{43})$ and $\Omega/2$ is the resonant Rabi frequency corresponding to a single beam. The terms in brackets take into account the differing dipole transitions between the relevant hyperfine levels in cesium ($F=4$, $F'\rightarrow 5$, 4, 3). The δ_{4j} are the corresponding detunings and, assuming equal populations of the Zeeman sublevels, the numerical values for the s_{4j} are $s_{45}=11/27$, $s_{44}=7/36$, and $s_{43}=7/108$. The function $f(t)$ represents the shape of the kicks, which in this work is close to rectangular: $f(t)=1$ for $0<t<\tau_p$ and zero otherwise. In the limit where $\tau_p\rightarrow 0$ we recover the DKR. The pulses repeat with period T. The dimensionless Hamiltonian for the kicked rotor is

$$H=\frac{\rho^2}{2}-k\cos(\phi)\sum_{n=1}^{N}f(t-n), \tag{2}$$

$H'=(4k_l^2T^2/m)H$; the primes are subsequently dropped. The classical stochasticity parameter is $\kappa=\Omega_{eff}\omega_R T\tau_p$, with $\omega_R=\hbar k_l^2/2m$ the recoil frequency. The quantum features of the DKR enter through the commutation relation $[\phi,\rho]=i\overline{k}$, where $\overline{k}=8\omega_R T$.

Approximately 10^5 Cs atoms are initially cooled in a magneto-optic trap (MOT) to a temperature of ~ 10 μK. The position distribution of the trapped atoms has a full width at half maximum of ~ 200 μm. The periodic potential is generated by a laser diode. The beam passes through an acousto-optic modulator, is collimated to a measured waist ($1/e^2$ intensity radius) of 520 μm, and is retroreflected to generate a one-dimensional potential. The Rabi frequency in the center of the MOT is $\Omega/2=310$ MHz. The finite widths of the beam waist and the atomic cloud entail a reasonably narrow distribution of κ, with RMS spread of 10% and $\kappa_{mean}\approx 0.9\kappa_{max}$, where κ_{max} is the kicking strength on the beam axis. In the following, when specifying κ this always refers to κ_{mean}. The pulse spacing used is $T=20$ μs ($\overline{k}=2.1$). In this study, we varied the pulse width (τ_p) and the detuning (δ). After trapping and cooling the atoms, the MOT is turned off leaving the atoms in the $F=4$ ground state. They have a 1:6 chance per spontaneous scattering to fall into the $F=3$ ground state, so in order not to lose these atoms we leave the repumping beam on during the experiment. This results in a small, but negligible heating [8]. To measure the atomic momentum distribution we use a

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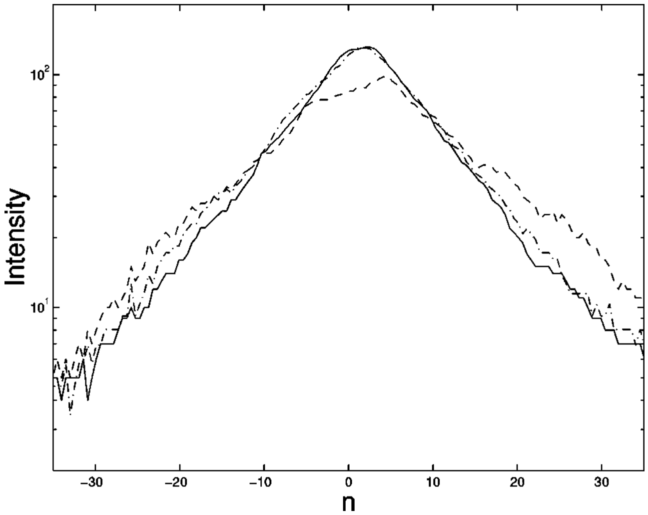

FIG. 1. The measured momentum distributions after 100 kicks for $\kappa=12.5$, $\bar{k}=2.1$ and spontaneous emission rate per kick of $\eta=0.76\%$ (solid), 2.3% (dot-dashed), and 4.6% (dashed). The corresponding detunings are $\delta_{45}/2\pi=4.0$, 1.3, and 0.6 GHz. The dimensionless integer momentum is defined as $n=p/2\hbar k_l$.

time-of-flight technique with a ''freezing molasses'' [4,8,9] and an expansion time of 12 ms.

We have also developed Monte Carlo wave-function simulations, which take into account the continuous distribution of recoil momenta along the axis defined by the kicking beams. The simulation also accounts for the finite duration of the kicking pulses. Spontaneous emission can occur at any time during the laser kick. This simulation was carried out by adding an interaction term $H_{int}=-\zeta u\bar{k}\phi\Sigma_{n=1}^{N}\delta(t-n)$ to the Hamiltonian (1), where ζ is either 0 or 1, $\langle\zeta\rangle=\eta$, and $u\bar{k}$ is the recoil momentum projected onto the kicking beam axis (u chosen randomly from the interval $[-1,1]$). The spontaneous emission recoil alters the ''numerical grid'' on which the coherent dynamics for the particular atom.

We previously reported on our measurements of the growth of the atoms' kinetic energy with time [8,9]. This was done for $\kappa=12.5$, $\bar{k}=2.1$, and detunings of $\delta_{45}=4.0$, 1.3 and 0.6 GHz, with corresponding spontaneous emission rates of $\eta=0.76\%$, 2.3%, and 4.6% per kick. A picture of the characteristic exponentially shaped momentum distribution was also presented for the $\delta_{45}=4.0$ GHz example [9]. The spontaneous emission introduces decoherence to the Q-DKR, which destroys dynamical localization and results in quantum diffusion [8], or momentum diffusion after the quantum break time. In Fig. 1 we present representative momentum distributions that we have measured after 100 kicks for the detunings of $\delta_{45}/2\pi=4.0$, 1.3 and 0.6 GHz. One can see that the line shapes remain essentially exponential in shape, which may appear contradictory, as exponential line shapes are the hallmark of dynamical localization. For our relatively small spontaneous emission rates delocalization emerges through energy diffusion and not a change in the character of the line shape. Experimental limitations prevent the measurement of line shapes for kick numbers in excess of ~100.

The numerical simulation results that can be directly compared to the experiments are presented in Fig. 2. The parameters used are the same as those quoted for Fig. 1. There is good agreement between the experiments and our simulations. The line shapes generated via the numerical analysis also show the exponential character.

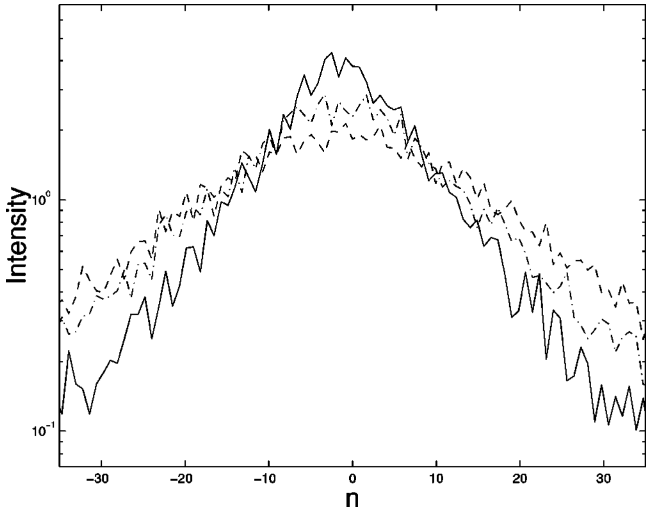

FIG. 2. Momentum distributions from a Monte Carlo wave-function calculation after 100 kicks for $\kappa=12.5$, $\bar{k}=2.1$ and spontaneous emission rate per kick of $\eta=0.76\%$ (solid), 2.3% (dot-dashed), and 4.6% (dashed).

In order to compare our results with the results of the Austin group [10,11] we present our numerical results on an extended scale, and also include the simulation results for a spontaneous emission rate of 13% (same values of $\kappa=12.5$ and $\bar{k}=2.1$); see Fig. 3. We see that for the 13% spontaneous emission rate the momentum distribution is beginning to bulge out. This is consistent with measurements from the Austin group [10,11]. Experimental studies that are able to observe the momentum distribution at large momenta will prove to be invaluable, as this appears to be the location where the character of the distribution first changes.

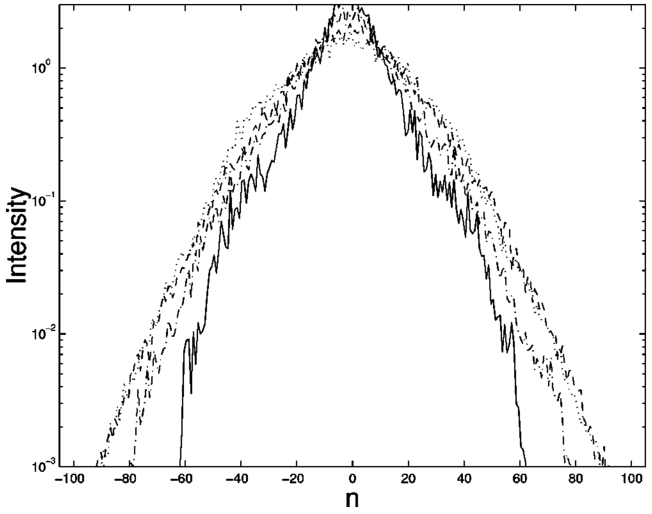

FIG. 3. Momentum distributions from a Monte Carlo wave-function calculation after 100 kicks for $\kappa=12.5$, $\bar{k}=2.1$ and spontaneous emission rate per kick of $\eta=0.76\%$ (solid), 2.3% (dot-dashed), 4.6% (dashed), and 13% (dotted).The dotted curve corresponds to the parameters used by the Austin group Refs. [10] and [11]. By viewing the simulated distributions out to higher momentum one can notice the emergence of nonexponential shape.

In summary, the atomic optics manifestation of the Q-DKR offers a unique and pristine environment for studying quantum chaos and decoherence. Much information about the system can be gained through measurements of the kinetic energy growth of the ensemble. Examination of the shape and character of the momentum distributions is also informative, and their observation will continue to be utilized in future quantum chaos experiments. For the spontaneous emission rates used in our experiment the line shapes remain essentially exponential. Monte Carlo wave-function simulations of our system support this conclusion. Similar theoretical results were found in the case of a modulated phase potential [12]. For higher spontaneous emission rates the exponential line shape shape degrades; this is consistent with the results from the Austin group [10,11] and the numerical simulations presented in this Brief Report. We have limited our present numerical study to parameters that can be achieved in current experimental set-ups, and interesting momentum distributions may occur in the presence of decoherence for sufficiently large kick numbers or high spontaneous emission rates. This will be addressed by our group in further studies.

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